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EFFICIENCY OF CROBEX AND CROBEX10 STOCK MARKET INDICES

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Abstract
The work of Haugen and Baker (1991) and Grinold (1992) has shown that market capitalisation-weighted indices are not mean-variance efficient. Further research by Amenc, Goltz, and Le Sourd (2006) proves that even naïve equal weighting can offer a better risk to return trade-off to investors in the developed markets. Based on earlier research findings of Zoricic, Dolinar, and Kozul (2014) and Dolinar, Zoricic and Kozul (2017) for the Croatian market which demonstrated that outperforming the cap-weighted index in an illiquid and undeveloped market is much more challenging the aim of this paper is to assess the efficiency of both CROBEX and CROBEX10 stock market indices. Efficient frontier was derived based on historical data ("ex post") for 5 revisions for each index. The distance from the efficient frontier was calculated revealing weaker efficiency but also greater diversification opportunities in the case of the broader CROBEX index. However, lower efficiency gains and higher estimation error in emerging market environment reduce significantly the out-of-sample potential for efficient index benchmarks. The analysis conducted in this paper makes it hard to assess if such potential truly exists but provides an insight based on calculation of indifference transaction costs following the work of Amenc et al. (2011).

Keywords: cap-weighted indices, index efficiency.

Jel Classification: G11; G12

INTRODUCTION

In the 1990's the research of Haugen and Baker (1991) and Grinold (1992) pointed out for the first time that stock market cap-weighted indices offer an inefficient risk to return trade-off. The finding was crucial both for practice and theory of the passive portfolio management as it corroborated a much earlier research of Roll (1977) who addressed the issue of the market portfolio in the Capital Asset Pricing Model (CAPM) being

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unobservable. Since it is unobservable a proxy has to be used in its place in order to apply the CAPM in practice. Roll argued that such a proxy would not represent a mean-variance efficient portfolio as opposed to the CAPM market portfolio which must be efficient if the CAPM assumptions hold (Amenc et al. 2006, 31).

According to Goltz and Le Sourd (2011) Roll's findings focused on the CAPM but in the broader context of the passive portfolio management they meant that there is no reason for any benchmark index (even if it is a proxy of the CAPM market portfolio) to be necessarily mean-variance efficient. However, the research of Amenc, Goltz and Le Sourd (2006) tested 11 indices from the equity markets of the US, Europe and Japan. They found that cap-weighted stock market indices are so inefficient that even naïve method of equal weighting almost always yields superior performance. The main cause seems to lie in the lack of diversification (especially in the case of broad-based indices) as the broader the index is the lesser the effect of many of its constituents when market cap-weighting is applied. Two main reasons behind the use of cap-weighting are the fact that such index can be easily implemented as a tradable portfolio (price changes do not lead to portfolio rebalancing) and that it was used by Sharpe (1964) to construct the CAPM market portfolio (Amenc, Goltz, and Le Sourd 2006, 35).

Implications of these findings for the portfolio management were huge as it turned out that efficient index benchmarks should be constructed. Various approaches have been pursued by researchers with the most promising ones being adopted by index providers. The performance of these benchmarks was analysed for instance in Amenc, Goltz, and Lodh (2012a) and Amenc et al. (2012b). The research of Amenc et al. (2014) also showed the growing interest of investment professionals for such investment opportunities and their growth potential. One line of research lead by Martellini (2008) and Amenc et al. (2011) even attempted the estimation of the CAPM market portfolio in the equity market. Building on this and other findings the improved efficiency also presents an opportunity to combine the newly proposed benchmark portfolios with factor investing approaches in order to provide systematic stock-picking strategies. Such possibility is explored in Amenc et al. (2016).

For the Croatian equity market the efficiency of CROBEX index was analysed by Zoricic, Dolinar, and Kozul (2014) and an attempt to estimate a more efficient portfolio was considered in Dolinar, Zoricic, and Kozul (2017). Both papers found that in a small and illiquid market outperforming a cap-weighted index seems to present a more challenging task than in developed markets. Ivanovic, Baresa, and Bogdan (2013) succeed in outperforming CROBEX only by adding cash funds as a proxy for risk-free asset and allocating assets between stocks and cash funds. Some findings even suggest that the construction of efficient portfolio should be based on a subset of CROBEX constituents concentrating in the most liquid stocks with largest market capitalisation – such as CROBEX10 constituents.

However, although the CROBEX index seems to be inefficient the paper by Zoricic, Dolinar, and Kozul (2014) lacked efficiency analysis based on index revision periods. This is even more important given the challenges of poor diversification opportunities (especially if the relatively small number of CROBEX constituents is further trimmed to 10) and unreliable data presented in the later work mentioned above. Therefore, in this research we analysed the efficiency of both CROBEX and CROBEX10 stock market indices deriving their respective efficient frontiers based on their constituents. Following the work of Amenc et al. (2006) the analysis was conducted "ex post" (based on historical data) for 5 revision periods revealing the maximum possible improvement in efficiency. Due to lower gains in efficiency and higher estimation error than in the developed markets it is hard to assess the potential for efficient index benchmarks in such an environment. However, based on the work of Amenc et al. (2011) calculation of indifference transaction costs was possible providing an insight since if the maximum possible improvement in efficiency cannot cover transaction costs an out-of-sample estimate of an efficient portfolio will certainly not cover them too.

1. METHODOLOGY

We applied the mean-variance optimisation originally proposed by Markowitz (1952) to derive a set of mean-variance efficient portfolios – the efficient frontier for each analysed data sample. The optimisation can be performed by minimizing the portfolio risk for a given level of portfolio return or by maximizing the portfolio return for a given level of risk. In our analysis we opted for the former in which case the optimisation problem is presented by the equations below (Amenc, Goltz, and Le Sourd 2006, 80).

$$\text{Minimize } \sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

Under the following conditions:

$$R_p = \sum_{i=1}^{N} w_i R_i, \tag{2}$$

$$\sum_{i=1}^{N} w_i = 1, \tag{3}$$

$$w_i \geq 0. \tag{4}$$

where, σ_p represents portfolio risk, σ_i represents the risk of asset i, σ_{ij} represents the covariance between the asset i and j, w_i and w_j represent portfolio weights, R_p refers to the portfolio return and R_i to the return of the asset i. Also, for the second term of Equation (1) relating to the covariance $i \neq j$ must hold.

Also, to measure the efficiency of the analysed indices their distance from the respective efficient frontiers needed to be measured. We adopted the approach taken by Amenc et al. (2006) who calculated the Euclidian distance for this purpose which is given by Equation (5):

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \tag{5}$$

where, D is the Euclidian measure of distance and the x and y represent two points in the coordinate plane.

2. DATA

The research was based on the CROBEX index, the oldest and largest (broad-based) index in the Croatian financial market introduced in 1997 by the Zagreb Stock Exchange (ZSE). It is not pure cap-weighted index given the free float adjustment for weights of its constituents and that each constituent's weight is further constrained to a maximum of 10%. Research also covered CROBEX10 index which includes 10 stocks from CROBEX index with the largest free float market capitalization and turnover. Both indices are price indices meaning that the dividends are not accounted for their calculation[2].

The research of this paper focused on the stocks that were listed on the ZSE and included in the CROBEX and CROBEX10 index in the period from September 2014 till March 2017. In the observed period indices composition was tracked based on 5 regular revisions. For CROBEX the number of the constituents varied from 23 to 25 stocks. A total number of 40 stocks were included in the index over the entire observed period. CROBEX10 had stable composition of 10 stocks. A total number of 13 stocks were included in CROBEX10 index over the entire observed period. Each revision presented separate sample since indices are subject to regular revision each six months.

Closing prices of the last working day of the week in which stock was traded were used to calculate the return based on price changes for the whole observed period. In case some days were non-working days or some stocks were not traded closing price of the preceding day was taken into account.

In order to assess the efficiency of indices mean-variance frontier of constituents of CROBEX and CROBEX10 indices was derived for each revision separately and the distance from the frontier was calculated for each index. Results are reported in the next section of the paper.

3. RESEARCH FINDINGS

By applying mean-variance optimisation to the collected data set we derived efficient frontier for 5 different samples (index revisions) for both (CROBEX and CROBEX10) indices. The results are (authors' calculation) presented in Figures 1 and 2. The performance of each market cap-weighted index was compared to the performance of the nearest efficient portfolio (minimal distance portfolio) in terms of the historical return and risk in Tables 1 and 2 (authors' calculation).

Figure 1 presents clearly that CROBEX index was inefficient which is indicated by its position on the graphs (below the efficient frontier of its constituents). In order to be able to compare the results to the results for the CROBEX10 index the minimal distance between the index and the efficient frontier portfolio was found as shown in Table 1. In addition, performance measures (return and risk) are shown for CROBEX and for the minimal distance portfolio from the efficient frontier for each sample.

[2] For more information please refer to: www.zse.hr

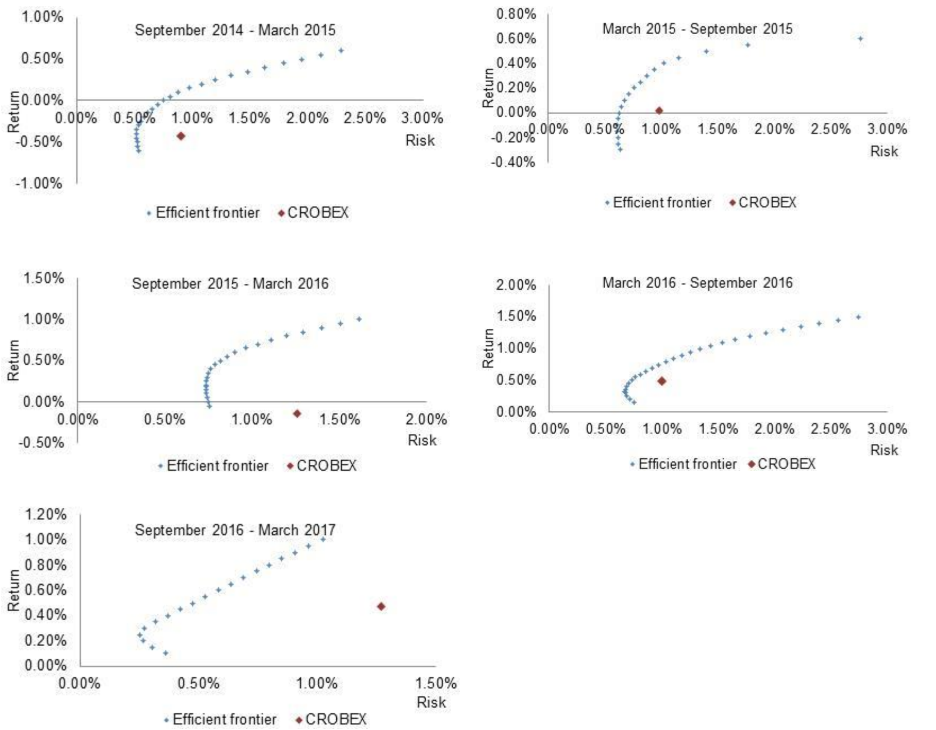

Figure 1. CROBEX position in comparison to efficient frontier of its constituents

Table 1. The minimal distance of CROBEX index from the efficient frontier of its constituents together with performance measures for the analysed samples

Sample	Minimal distance from efficient frontier	Rank	CROBEX		Portfolio from efficient frontier	
			Return	Risk	Return	Risk
September 2014–March 2015	0.003860	3	-0.43%	0.90%	-0.40%	0.52%
March 2015–September 2015	0.002935	2	0.01%	0.99%	0.20%	0.76%
September 2015–March 2016	0.006170	5	-0.14%	1.26%	0.19%	0.74%
March 2016–September 2016	0.002167	1	0.49%	1.00%	0.65%	0.86%
September 2016–March 2017	0.005652	4	0.47%	1.27%	0.90%	0.91%

The average minimal distance of CROBEX index from the efficient frontier of its constituents for five samples presented above was 0.004157. The analysis presented in the Table 1 reveals that CROBEX showed the highest efficiency during the March – September 2016 period accompanied also by the solid performance in the March to September 2015 revision. The poorest performance was related to the September 2015 – March 2016 period. Presented results confirmed that mean-variance optimization weighting of index constituents produced significantly more efficient portfolios which dominated the cap-weighted index either by higher return and lower risk.

The same efficiency test has been performed for CROBEX10 index.

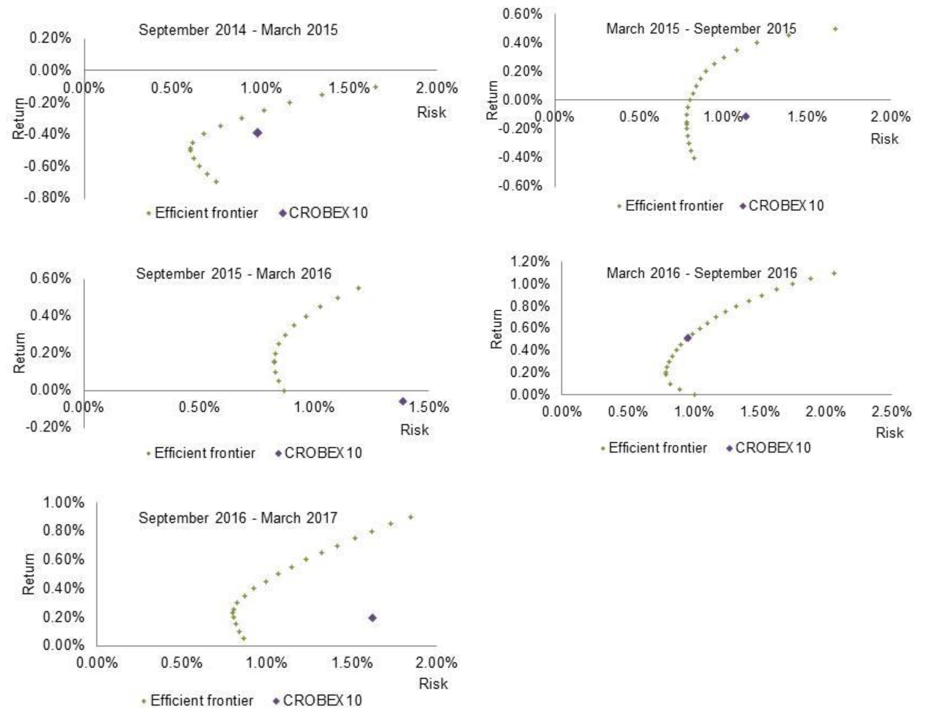

Figure 2. CROBEX10 position in comparison to efficient frontier of its constituents

Figure 2 shows results similar to CROBEX index except in the case of the March – September 2016 sample in which case the CROBEX10 index was almost on the efficient frontier. Again, the minimal distance between the index and the efficient frontier of its constituents was found and shown together with performance measures (return and risk).

Table 2. The minimal distance of CROBEX10 index from the efficient frontier of its constituents together with performance measures for the analysed samples

Sample	Minimal distance from efficient frontier	Rank	CROBEX10		Portfolio from efficient frontier	
			Return	Risk	Return	Risk
September 2014–March 2015	0.001310	2	-0.39%	0.98%	-0.30%	0.89%
March 2015–September 2015	0.003486	3	-0.12%	1.14%	-0.05%	0.79%
September 2015–March 2016	0.006003	5	-0.06%	1.39%	0.15%	0.83%
March 2016–September 2016	0.000172	1	0.51%	0.95%	0.50%	0.94%
September 2016–March 2017	0.005513	4	0.19%	1.63%	0.70%	1.42%

CROBEX10 showed the highest efficiency in March – September 2016 period while the poorest performance was achieved in period September 2015 – March 2016. It can be noticed that the same revision periods related to the highest and lowest efficiency of the CROBEX index also. However, in comparison to CROBEX, minimal distances from the efficient frontier for CROBEX10 were less (with the average distance standing at 0.003297) which led to the conclusion that CROBEX10 presented a more efficient index. Contrary to what might be expected better efficiency was actually caused by fewer diversification opportunities offered by narrow-based CROBEX10 index. Therefore the efficient frontier of CROBEX10 was actually reduced compared to the efficient frontier of CROBEX. In Figure 3 this is presented even without plotting the efficient frontier of CROBEX10 as the CROBEX10 was clearly positioned much farther from the efficient frontier of CROBEX than CROBEX itself. These findings correspond to the findings by Amenc et al. (2006) for the US, European and Japanese stock markets. However, the problem in the case of Croatian stock market arose from the fact that CROBEX index wasn't that much broader than the CROBEX 10. In fact in the developed markets it would be considered narrow-based itself.[3] Also, the composition of the efficient portfolios for which performance was stated in Tables 1 and 2 reveals that these portfolios were highly concentrated in a few stocks. Much more so than the cap-weighted indices they were based on and in relative terms even more so in the case of the minimal distance efficient portfolio for CROBEX. This is contrary to the main advocated principle of risk reduction and efficiency improvement through diversification[4] and could therefore be argued that works well only in-sample. Out-of-sample such concentration could rather imply the opposite — taking on more risk. Therefore, additionally in order to ensure that all index constituents were included in the index and that deconcentration was increased constraints on weight were imposed as proposed in Amenc et al. (2011) in the optimisation process. Flexibility parameter lambda (λ) was introduced which imposes constraints on weights depending on the number of constituents (N) in the index. Thus, resulting in the same effect of constraints on the relative scale regardless of the index size.[5] Using lambda the constraint on the minimal weight was imposed by $1/(\lambda N)$ and on the maximum by λ/N.

The efficient frontier was derived using the values λ=2 and λ=4 for the March – September 2015 sample for the CROBEX constituents. This particular sample was chosen as it presented the median distance rank of CROBEX from its efficient frontier. The results are plotted in the Figure 3 (author's calculation).

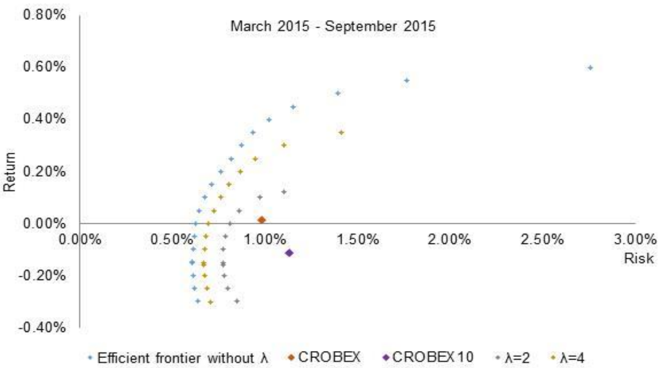

Figure 3. CROBEX and CROBEX10 position in comparison to efficient frontier of CROBEX constituents with imposed constraints λ=2 and λ=4

[3] As reported in the data section the number of constituents for CROBEX varied from 23 to 25 stocks in the analysed period which is less than Dow Jones 30, DAX 30 or CAC 40.

[4] Such efficient portfolios actually resemble much more the cap-weighted portfolios of broad-based indices in the developed markets. According to Amenc et al. (2006, 8): "…if an index has more than 500 components, 90% of the components make up an almost negligible part of the index weights."

[5] The portfolio weight of 10% does not represent the same size in relative terms in a portfolio of 500 and of 10 stocks. Parameter lambda has taken this into account.

Figure 3 reveals interesting results. For λ=2 minimum weight of constituents was 2% and maximum stood at 8% while for λ=4 minimum weight was 1% and maximum stood at 16%. It could be noticed that the efficient frontier was the widest without lambda i.e. without imposing constraints. When lambda was set to two higher constraints were imposed and the efficient frontier therefore shrank significantly. On the other side the deconcentration of the minimal distance efficient portfolio, calculated as in Strongin, Petsch, and Sharenow (2000) as the effective number of stocks in the index, rose significantly (from cca. 7 without constraints to cca. 17). Obviously, a lot of attention has to be devoted to the trade-off between the increase in the deconcentration of portfolio and the reduction in investment opportunities in future research efforts.

Also, since the research findings were based on the in-sample fit they did not allow the assessment of the potential for efficient index benchmarks in illiquid and undeveloped Croatian market generally. As already mentioned in the introduction the main concern in this respect was that due to lower efficiency improvement in out-of-sample estimation (even in the case of naïve equal weighting) and greater estimation error in such environment the free-float adjusted market cap-weighted may never be outperformed. However, in order to try to address this issue using the presented findings we computed the indifference level of transaction costs for the whole observed period based on the work of Amenc et al. (2011). This measure indicates the level at which the transaction costs would neutralize the return difference with CROBEX and CROBEX10 respectively.

Table 3. Turnover, indifference level of transaction costs and concentration[6]

Portfolio	Annual one-way turnover	Excess turnover vs. cap-weighted indices[7]	Annual return difference over cap-weighted indices	Indifference level of transaction costs
Efficient portfolio based on CROBEX	225.92%	174.96%	14.44%	8.25%
CROBEX	50.96%	0.00%	-	-
Efficient portfolio based on CROBEX10	194.58%	161.47%	10.25%	6.35%
CROBEX10	33.11%	0.00%	-	-

Indifference level of transaction costs as presented above was relatively high. Therefore, it can be concluded that it is unlikely that an investor would ever pay such high costs even in the illiquid market. However, the values presented in the Table 3 refer to the minimal distance efficient portfolios which do not represent an efficient benchmark typically estimated out-of-sample. The turnover data stated in the table may differ for such portfolios. We found for the analysed data set that for instance the minimum variance portfolio (based on CROBEX index) exhibited an annual one-way turnover of over 240%. This is a slight increase in respect to 225%, but the true problem with the minimum variance portfolio in the context of this analysis is that it was not supposed to dominate the CROBEX return-wise. In the analysed period this was just the case yielding negative annual return difference over cap-weighted indices. However, we argue that this does not render the analysis presented (author’s calculation) in the Table 3 futile as it is clear that even from this aspect CROBEX should serve as a base for

[6] CROBEX and CROBEX10 performances were compared to their efficient portfolio. In this case efficient portfolio was related to all minimal distance portfolios from the efficient frontier of their constituents taken together for the whole observed period on annual basis for CROBEX and CROBEX10 separately.

[7] Cap-weighted indices are CROBEX and CROBEX10.

attempts to construct and estimate efficient portfolios. Moreover, the indifference level of transaction costs of over 8% in the case of CROBEX seems to indicate a small yet existing margin for such estimation efforts.

CONCLUSION

Inefficiency of the market cap-weighted indices is well documented. Research for the developed markets demonstrated that even naïve equal weighting resulted in a significant efficiency improvement over cap-weighting. However, research for the Croatian market showed that in the case of illiquid and undeveloped markets it can be much more difficult to outperform the cap-weighted index. Some results even suggested that focusing on a smaller number of the most liquid stocks with the largest market capitalisation could yield better results than the estimation of a more efficient index based on the constituents of CROBEX (a broad-based index).

Therefore, in order to concentrate future research efforts this research provides an assessment of efficiency of both CROBEX and CROBEX10 indices based on historical data presenting the maximum possible improvement in efficiency for each index revision. The results showed inefficiency of both indices but also greater efficiency of a narrow-based CROBEX10 index which implies, however, a lower potential for efficiency improvement due to fewer diversification opportunities. This is in line with the research results for the developed markets the only problem being the relatively small number of constituents even in the case of broad-based CROBEX index. Therefore, the efficient in-sample portfolios were heavily concentrated in order to exploit the risk-return characteristics of a few stocks. Since such improvement in efficiency does not rely on improvement through diversification but could rather imply taking on more risk, especially in the out-of-sample estimation, the efficient frontier of CROBEX index was derived by applying constraints to the portfolio weights. Results showed the high sensitivity to the level of constraints imposed allowing for significant deconcentration effects but at the expense of significant reduction in the efficient frontier.

Also, since the research findings were based on the in-sample fit they did not allow the assessment of the potential for efficient index benchmarks in illiquid and undeveloped Croatian market generally. The general concern being that due to lower efficiency improvement in out-of-sample estimation (even in the case of naïve equal weighting) and greater estimation error in such environment the free-float adjusted market cap-weighted may never be outperformed. However, the research findings provided a calculation of indifference level of transaction costs which standing at above 8% for the closest efficient portfolio for the CROBEX index suggested a small but existing margin for estimation error.

REFERENCES

Amenc, Noel, Felix Goltz, and Veronique Le Sourd. 2006. *Assessing the Quality of Stock Market Indices: Requirements for Asset Allocation and Performance Measurement*. Nice: An EDHEC Risk & Asset Management Research Centre Publication

Amenc, Noel, Felix Goltz, and Ashish Lodh. 2012a. Choose Your Betas: Benchmarking Alternative Equity Index Strategies. *The Journal of Portfolio Management* 39 (1): 88–111.

Amenc, Noel, Felix Goltz, Ashish Lodh, and Lionel Martellini. 2012b. Diversifying the Diversifiers and Tracking the Tracking Error: Outperforming Cap-Weighted Indices with Limited Risk of Underperformance. *The Journal of Portfolio Management* 38 (3): 72–88.

Amenc, Noel, Frederic Ducoulombier, Felix Goltz, Ashish Lodh, and Sivagaminathan Sivasubramanian. 2016. Diversified or Concentrated Factor Tilts? *The Journal of Portfolio Management* 42 (2): 64–76.

Amenc, Noel, Felix Goltz, Ashish Lodh, and Lionel Martellini. 2014. Towards Smart Equity Factor Indices: Harvesting Risk Premia without Taking Unrewarded Risks. *The Journal of Portfolio Management* 40 (4): 106–122.

Amenc, Noel, Felix Goltz, Lionel Martellini, and Patrice Retkowsky. 2011. Efficient Indexation: An Alternative to Cap-Weighted Indices. *Journal of Investment Management* 9 (4): 1–23.

Dolinar, Denis, Davor Zoricic, and Antonija Kozul. 2017. Towards the Estimation of an Efficient Benchmark Portfolio: The Case of Croatian Emerging Market. *Zagreb international review of economics & business* Special Conference Issue (April): 13–24.

Goltz, Felix, and Véeronique Le Sourd. 2011. Does Finance Theory Make the Case for Capitalisation-Weighted Indexing? *The Journal of Index Investing* 2 (2): 59–75.

Grinold, Richard C. 1992. Are Benchmark Portfolios Efficient? *The Journal of Portfolio Management* 19 (1): 34–40.

Haugen, Robert A., and Nardin L. Baker. 1991. The Efficient Market Inefficiency of Capitalization-Weighted Stock Portfolios. *The Journal of Portfolio Management* 17 (1): 35–40.

Ivanovic, Zoran, Suzana Baresa, and Sinisa Bogdan. 2013. Portfolio optimization on Croatian capital market. *UTMS Journal of Economics* 4 (3): 269–282.

Markowitz, Harry M. 1952. Portfolio Selection. *The Journal of Finance* 7 (1): 77–91.

Martellini, Lionel. 2008. Towards the Design of Better Equity Benchmarks: Rehabilitating the Tangency Portfolio from Modern Portfolio Theory. *The Journal of Portfolio Management* 34 (4): 34–41.

Roll, Richard. 1977. A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory. *Journal of Financial Economics* 4 (2): 129–176.

Sharpe, William F. 1964. Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance* 19 (3): 425–442.

Strongin, Steve, Melanie Petsch, and Greg Sharenow. 2000. Beating benchmarks. *The Journal of Portfolio Management* 26 (4): 11–27.

Zoricic, Davor, Denis Dolinar, and Antonija Kozul. 2014. The Market Index Benchmark and Adequate Compensation for Systematic Risk in an Illiquid and Undeveloped Financial Market. In *Risk Management: Strategies for Economic Development and Challenges in the Financial System*, ed. Danijela Milos Sprcic, 257–277. New York: Nova Science Publishers, Inc.